

Program GEOCON
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User Guide

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1. Introduction

GEOCON performs three-dimensional coordinate transformations between the NAD 83(HARN) coordinates and the NAD 83(NSRS2007) coordinates. GEOCON also issues information about the quality of the transformation at each point, and notifications regarding poor quality results.

This document (“The User Guide”) is a companion to the GEOCON Operating Instructions. Please refer to the latter document for details on core elements, such as formats. It may be helpful to read the Operating Instructions before reading this guide.

2. Coordinate Transformation

The coordinate sets, both HARN and NSRS2007, are defined quantities (as of a particular date). Good or bad, the coordinate differences between the HARN and the NSRS2007 are constants. There can be no expectation of smoothness in a coordinate transformation. Plots of high resolution coordinate transformations may not look pleasing. Blunders in the older coordinates that are corrected in the newer coordinates will appear as abnormal shifts. But, they are not to be excluded from consideration. They are actual differences between two disseminated coordinate sets. Further, abnormal coordinate differences exist because some errors were corrected when making the newer coordinate set.

To support our user communities, NGS decided to create a coordinate transformation between the NAD 83(HARN) and the NAD 83(NSRS2007). This, in turn, implies seeking a mathematical mapping between the two coordinate sets, *irrespective of their values*.

3. Coordinate Differences: Between the Points

One could argue that a coordinate transformation only exists at the points that define the coordinate differences. While being a legitimate argument, such a perspective would provide no guidance on how to treat intervening points that were directly or indirectly tied to the defining coordinates. For guidance we must consider field practice. And, this practice will not just include surveyors, but all practitioners who create georeferenced data sets.

It is natural to define the coordinate transformation between defining points as being an intermediate value. As such, one does not want to see extraneous oscillations in a transformation function, even when there are large, local excursions at the defining points. In essence, one wishes to “connect the dots”.

To satisfy the needs of honoring the data and generating intermediate values, we select a gridding method of splines in tension (Smith and Wessel, 1990). This models the physical behavior of a thin, flexible plate that passes through the defining points. However, such a model, by itself, is subject to overshoots and undershoots when data differences occur near gaps in irregularly spaced data. By mathematically applying

tension at the edges of a grid, it is possible to suppress the oscillations, and generate representative intermediate values. For GEOCON, after some tests, a 1' x 1' grid with a tension parameter of 0.4 was selected.

Basically, the fitted grid is modeling the coordinate differences a practitioner would obtain when performing two different least squares adjustments of the same survey data when controlled by two different control point coordinate sets. Or, the grid models the differences for photogrammetric data, or synthetic aperture radar, or LIDAR, or any other coordinate measurement system that ties to the control point coordinate set.

If, however, a practitioner only performs a single point tie, then all of the geospatial data should be transformed by the unique coordinate difference of the source control point.

Consider an extreme example; the ellipsoidal height for the point M 123 (PID = TT2413) as obtained from the NGS database in November 2011 (Appendix A.1.):

| | | |
|--------------------------------------|----------------|------------|
| NAD 83(HARN) ellipsoidal height: | 486.945 meters | (06/20/05) |
| NAD 83(NSRS2007) ellipsoidal height: | 641.786 meters | (07/17/09) |

The change of nearly 155 meters is due to the re-measurement and readjustment of M 123.

Now suppose a practitioner has some regional data in a HARN coordinate set. The data are tied to the abnormal point M 123 and other nearby control points with much smaller coordinate differences. The ellipsoidal height coordinate differences will get larger and larger for the data points nearer and nearer M 123. On the other hand, if the practitioner connected the work solely to M 123, then *every* ellipsoidal height must be increased by exactly 154.841 meters irrespective of the distance from M 123.

This is one reason why the National Geodetic Survey considers actual recomputation of geospatial data, and not coordinate transformations, as “best practice”. The coordinate transformation is, at its heart, only a model of actual geospatial measurement and processing.

4. Coordinate Differences: Inside the Cell

The analysis of the NAD 83(NSRS2007) National Readjustment (Milbert 2008) showed a number of surprising results. One was the highly local character of the network. Over 50% of all GPS vectors were 31 km or less in length. Figure 5.3 of Milbert (2008, pg. 12) is reproduced below as Figure 4.1. Note the significant number of GPS vectors of just 1 or 2 km in length.

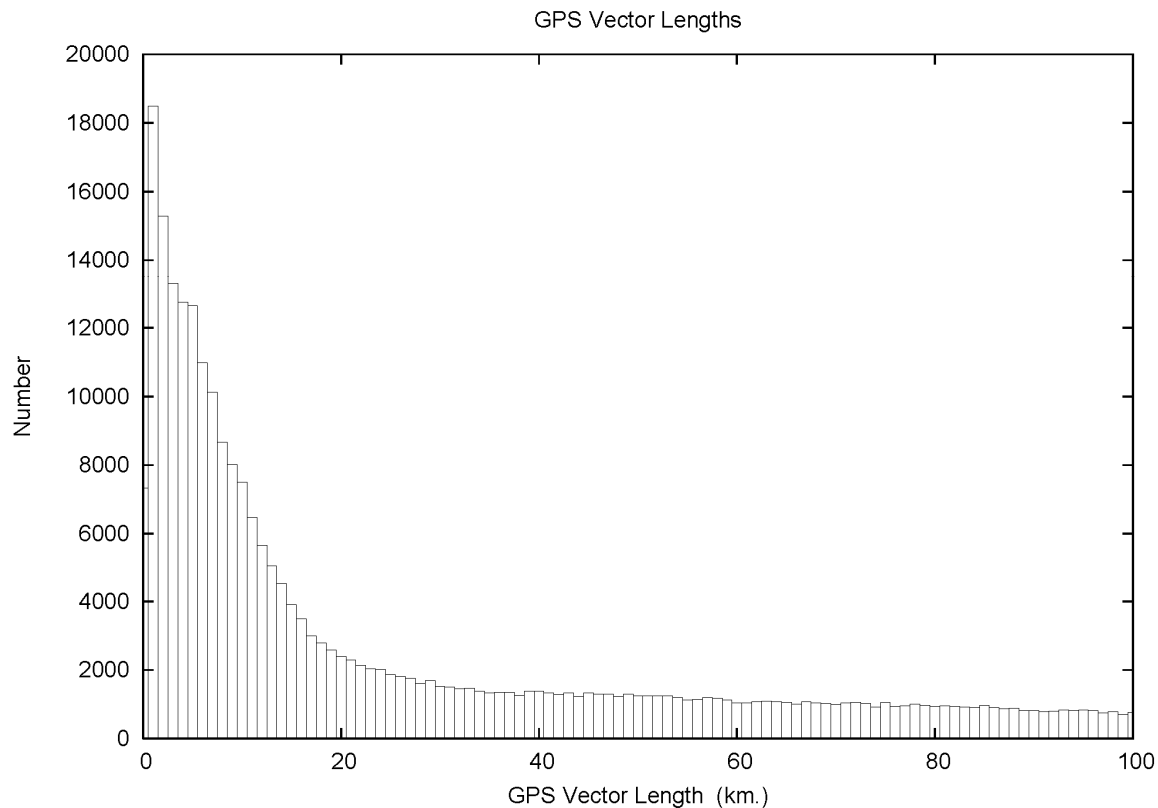


Figure 4.1. Distribution of GPS vector lengths, 0 to 100 km. 1 km bin size.

Even though a grid size of 1' x 1' was selected to model the coordinate differences, it can be expected that there will be clusters of defining points at some of the grid nodes. This is because of the close spacing of some of the control points. This naturally raises a question. How can one “honor the data” for multiple coordinate differences near a grid node?

It was decided to select the most representative control point by means of a median filter. The median value of a set is the value that delimits the higher half and lower half of the set. It can be obtained by sorting the set and choosing the middle value. In the case of an even number of points, the mean of the two central points is reported. The advantage of the median is that it is a robust estimate when more than two data are present. An outlier does not disturb the median.

To further increase the robustness, a modified median procedure was used prior to gridding. In the case of a cluster of exactly 2 points, a search was performed in the 1' to 2' ring surrounding the central 1'x1' cell. If the search found a point, it was used as a tiebreaker to select the winning median point in the central cell. If the 1' to 2' ring was insufficient, then the 2' to 3' ring was searched for a tiebreaker. If, after two ring searches no tiebreaker was found, then the central cell median was selected at random. It was found that that the ring search procedure was able to reduce 8177 pairs to 1534 pairs. And, by using a random selection, the possibility of the median being influenced by an abnormal point in the pair is additionally halved.

The philosophy in choosing a median procedure is rooted in the likely practice of geospatial professionals. In the presence of a cluster of control points, connections

should be made to a sufficient number to confirm the connection to the control network. Depending upon the accuracy of the positioning measurements, significant discrepancies may be identified. And, ties to suspect control points would be discarded. This is standard practice in surveying.

5. Transformation Quality

The preceding two sections illustrate the challenge in describing the quality of coordinate transformation.

It is possible that georeferenced data may be tied to control points with both standard coordinate differences and abnormal coordinate differences.

It is possible that georeferenced data may be tied solely to control points with standard coordinate differences, but be near points with abnormal coordinate differences.

Conversely, it is possible that georeferenced data may be tied solely to control points with abnormal coordinate differences, but be near points with standard coordinate differences.

Further, there can be highly variable scale. “Near” may refer to spacing of a few hundred meters or a few hundred kilometers. “Standard” and “abnormal” may refer to coordinate differences of less than a millimeter to over 100 meters. User accuracy requirements may vary from millimeter to multi-meter network accuracy, and may address only horizontal or only vertical components.

To represent transformation quality, a statistical resampling procedure known as cross-validation was selected. Cross-validation is useful in assessing predictive models (Efron and Tibshirani, 1998). It is appropriate to consider coordinate transformation as an exercise in prediction. The NGS database of coordinates with multiple coordinate realizations represents prior knowledge. Other practitioners, whose data are never known to NGS, will establish coordinates traceable to the NGS database. We seek to predict the transformed coordinates from those unknown coordinates.

In its simplest form, cross-validation consists of cutting a data set in half. Call the first half the *training set*, and build the prediction model from the first half. Then compare the second half of the data, called the *validation* or *testing set*, to the model predictions. Similarly, one may exchange the two data halves, and repeat the process.

In the extreme case, one can imagine taking a set of data, withholding a single data point, producing a unique model, and then computing the difference between the single withheld data point and the model. And, one can imagine doing this sequentially for every single point in the data set. This method is known as the jackknife (Efron 1979).

Needless to say, the jackknife can entail a large computational burden for sizable data sets. However, there is a middle ground. One can compute K-fold cross-validation (Efron and Tibshirani, *ibid*). The data set is partitioned into K subsets. In sequence, each

subset is designated as a testing set, and is temporarily withheld from the data set. The model is computed from each reduced data set, and differences are computed between the temporarily withheld data and the model prediction. The process is sequenced K times until a prediction error is established for each data point.

For GEOCON 69-fold cross-validation was performed. The master data set was 69540 point pairs. So each testing subset was a little over 1000 points. This means 69 training grids were computed for each component, for the regions of CONUS, Alaska, and Puerto Rico/Virgin Islands. For example, the 207 training grids for CONUS took about 8 hours to compute. The result was a set of cross-validation errors for the coordinate differences in latitude, longitude, and height.

The appeal of cross-validation error is that it quantifies the abnormality of a point. Consider the case of M 123. The ellipsoid height coordinate difference (2007 - HARN) is +154.797 meters. The cross-validation error is +154.8192 meters, showing that the point is definitely abnormal when compared to its neighbors.

By also gridding the cross-validation error as a quality measure, it is possible to quantify the uncertainty associated with varying field procedures and processing. If one is in the midst of normal control points, then the cross-validation errors will tend to be small. As one approaches an abnormal point, the quality measure will get larger, indicating increasing uncertainty about the proper coordinate transformation value to use.

For the case of a cluster of points, it is important to reflect the possibility that one of the points may be abnormal. Recall, when gridding the coordinate differences, a modified median procedure was used to select the points. Such a procedure will drop abnormal points in clusters. For the quality grids, the median is not used. Rather the worst case cross-validation error in a cluster is gridded. The worst case is selected by choosing the error that is furthest in magnitude from the median error (an “anti-median”).

In this way, the coordinate transformation grids of GEOCON provide the most likely transformations to apply to geospatial data. And, the transformation quality grids give a conservative idea of the quality of the transformation.

6. An Example in Northeast Colorado

It is useful to consider some raw coordinate data and its conversion into grids as illustration. Consider a region in northeast Colorado (Figure 6.1):

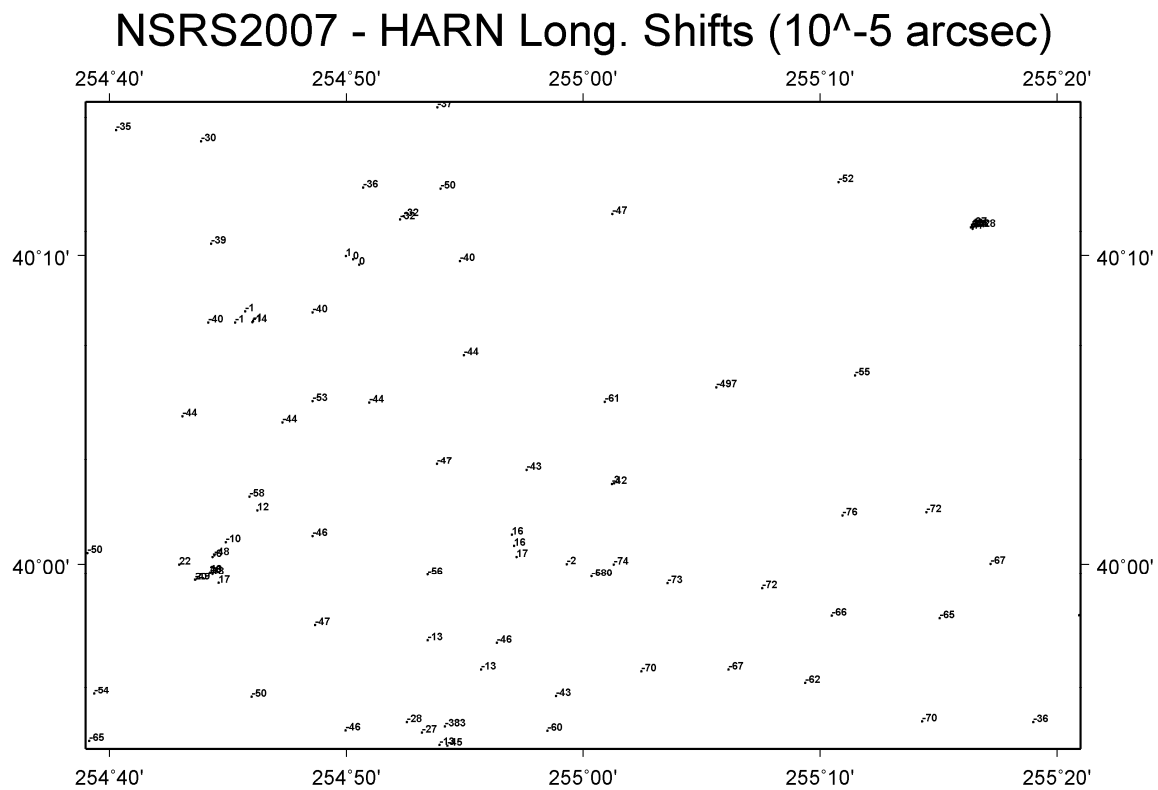


Figure 6.1. Longitude Coordinate Differences, Northeast Colorado.

To aid in discussion, the permanent identifiers (PID) of the points are displayed in Figure 6.2. The values portrayed in Figure 6.1 are the coordinate differences in longitude taken in the sense of NAD 83(NSRS2007) – NAD 83(HARN). In computing the shifts, the sense of the longitudes are positive East. The units are 0.00001 arc seconds. Thus, the numerical values in Figure 6.1 (as well as in subsequent figures) are integers. The small dots present near each number are the locations of the points. The small dots are *not* decimal points.

Considerable variation in the coordinate longitude shifts is seen in Figure 6.1. However a nominal shift of -0.00050 arc seconds is evident. Close inspection of the shifts will show some abnormal values. In some cases, the points are clustered so tightly, it is impossible to resolve the numbers.

| PID | Long. Shift (0.00001 arc sec) | Long. Shift (cm) |
|--------|----------------------------------|---------------------|
| KK2067 | -333 | -7.8 |
| KK2064 | -580 | -13.7 |
| LL1240 | -497 | -11.8 |
| LL1465 | +2328 | +55.1 |

Select NSRS2007 - HARN Long. Shifts (10^{-5} arcsec)

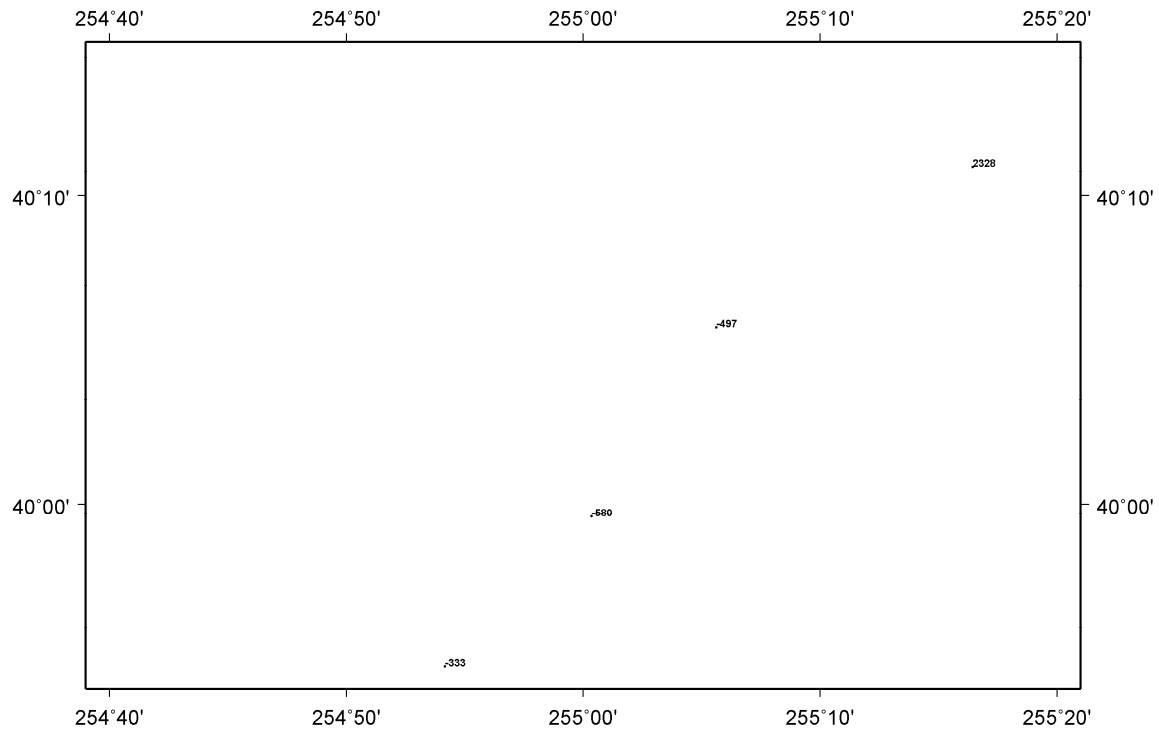


Figure 6.3. Abnormal Longitude Coordinate Differences, Northeast Colorado.

Select NSRS2007 - HARN PID's

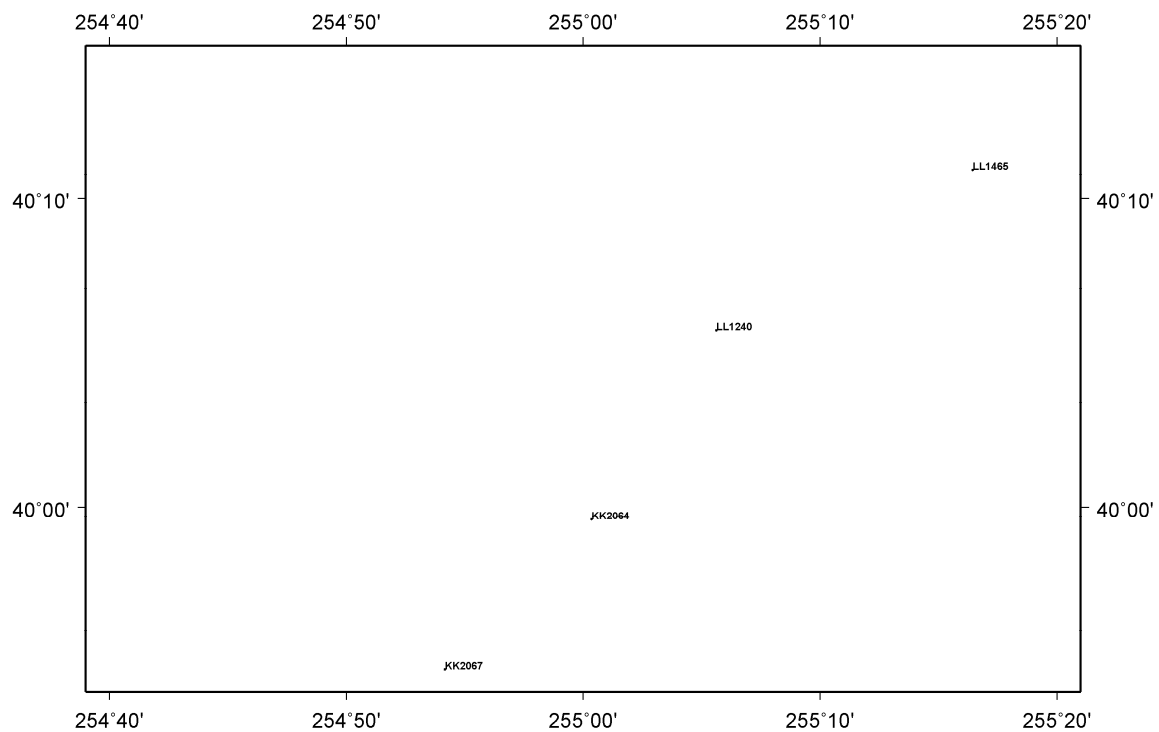


Figure 6.4. Permanent Identifiers (PID) of Abnormal Longitude Differences.

When the data in Figure 6.1 are gridded, we obtain the results portrayed in Figure 6.5. Once again, the units are 0.00001 arc seconds, the values displayed are integers, and the small dots represent grid nodes, and *not* decimal points.

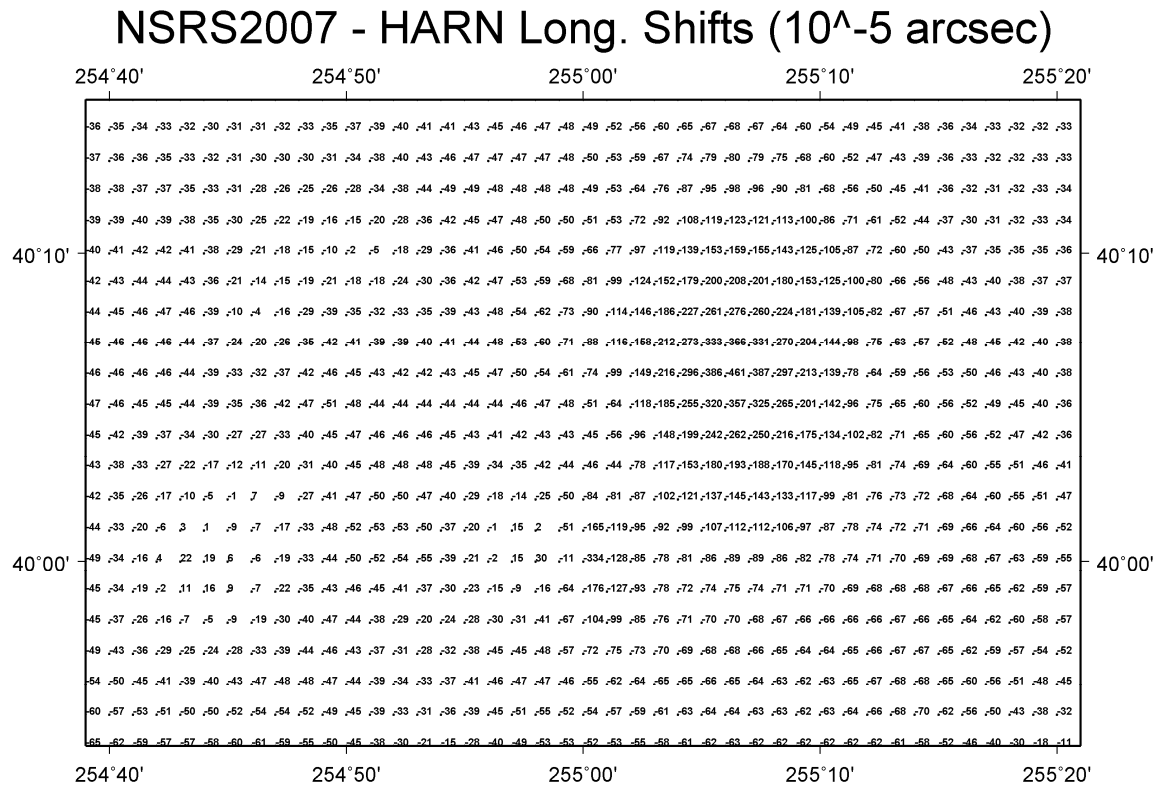


Figure 6.5. Gridded Longitude Coordinate Differences, Northeast Colorado.

We note that 1' x 1' grid captures the general magnitude of the coordinate shifts. It models the smaller shifts in the vicinity of 40° 00' N, 254° 45' E. Two of the four abnormal shifts (KK2064 and LL1240) are also evident as depressions in the grid. In particular, it is seen that the depression formed by LL1240 extends for some distance in all directions. This is because there are no neighbors nearby.

The use of a grid model will perform well if, for example, a local survey connects to LL1240 in addition to its neighbors: AE6472, AB3291, AB3295, AB3294, etc. One can see that a survey that solely connects to AE6472 (-0.00055 arc sec), but does not connect to LL1240 (-0.00497 arc sec), has a nominal longitude shift of -0.00055 arc seconds. But as those local survey points approach LL1240, they will get progressively worse shifts. This situation must be reported to the coordinate transformation user. This is done by means of the transformation quality grid.

In addition, while two of the abnormal shifts were modeled in the coordinate transformation grid, two other shifts were not (KK2067 and LL1465). These points were in clusters within the 1' x 1' grid resolution. They were dropped by the modified median filter prior to the actual gridding. The points are plotted in Figure 6.6.

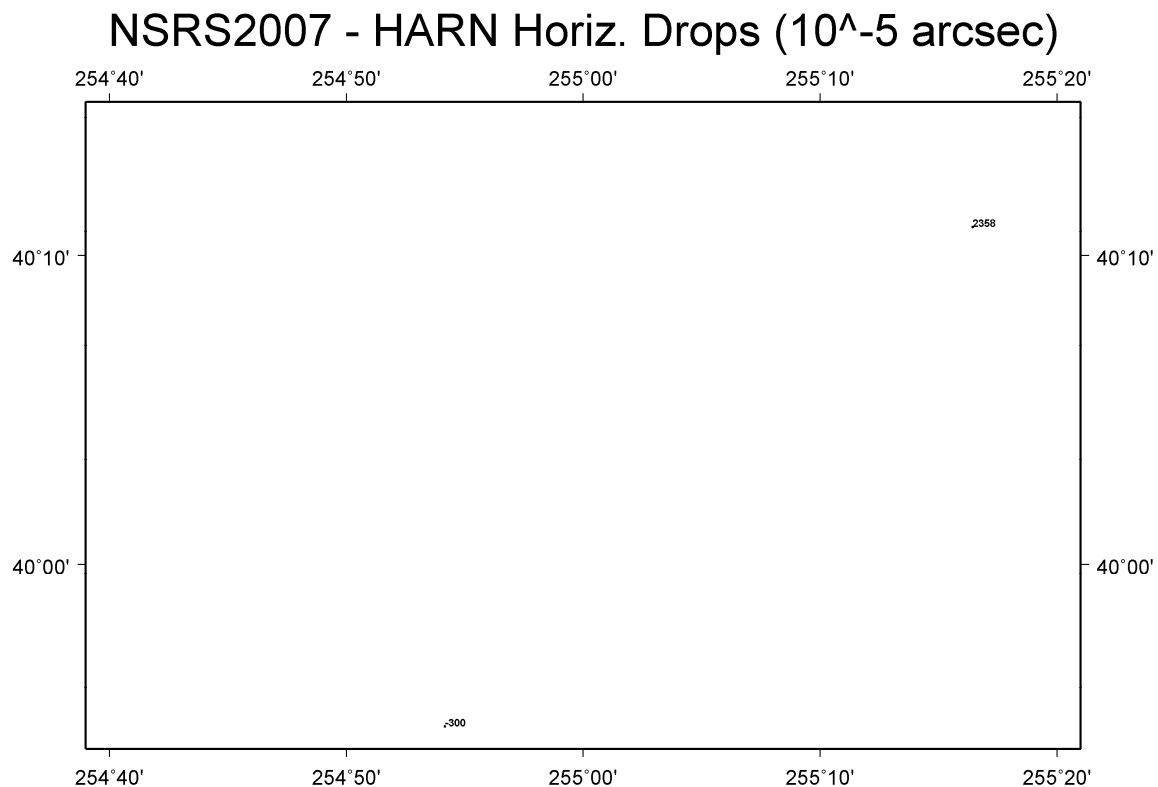


Figure 6.6. Abnormal Longitude Differences Dropped by Median Filter.

Note that the values in Figure 6.6 are not the actual longitude coordinate differences. Rather, they are the difference between the abnormal longitude differences of Figures 6.1 and 6.3, and the gridded longitude differences of Figure 6.5. Also note that the gridded values in Figure 6.5 are undisturbed by these abnormalities.

Once again, these abnormal quantities must be communicated to the coordinate transformation user. It is assumed that local surveying, mapping, and other geospatial work will have made multiple connections to nearby points, and have detected the issues. So the filtered transformation grid represents the most likely situation. As discussed earlier, the situation is reported through the transformation quality grid.

Figure 6.7 plots the cross-validation errors in the sense of measured difference – gridded difference, where the grid was obtained after dropping that specific measured difference (and around 1000 other randomly selected points throughout the entire data set). As before, units are 0.00001 arc seconds, and longitudes are positive East. To assist in inspection, the cross-validation errors of the four abnormal points are shown in Figure 6.8.

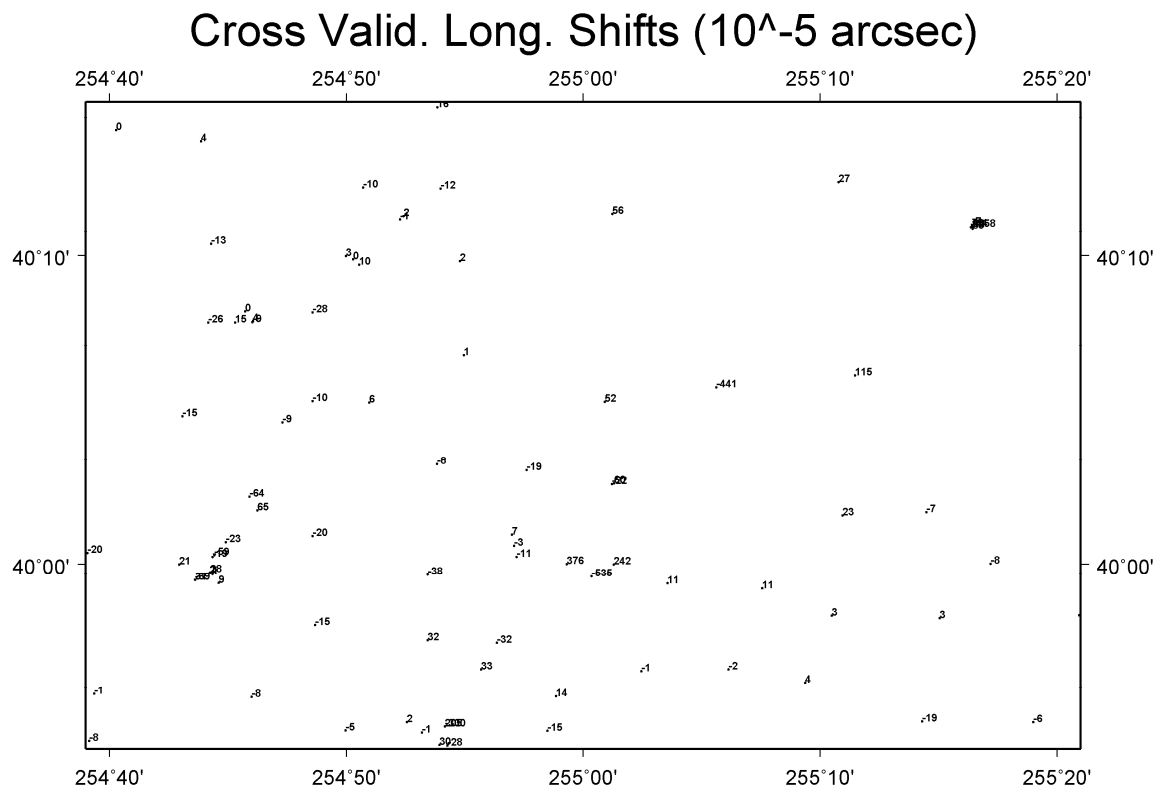


Figure 6.7. Cross-Validation Error of Longitude Coordinate Differences.

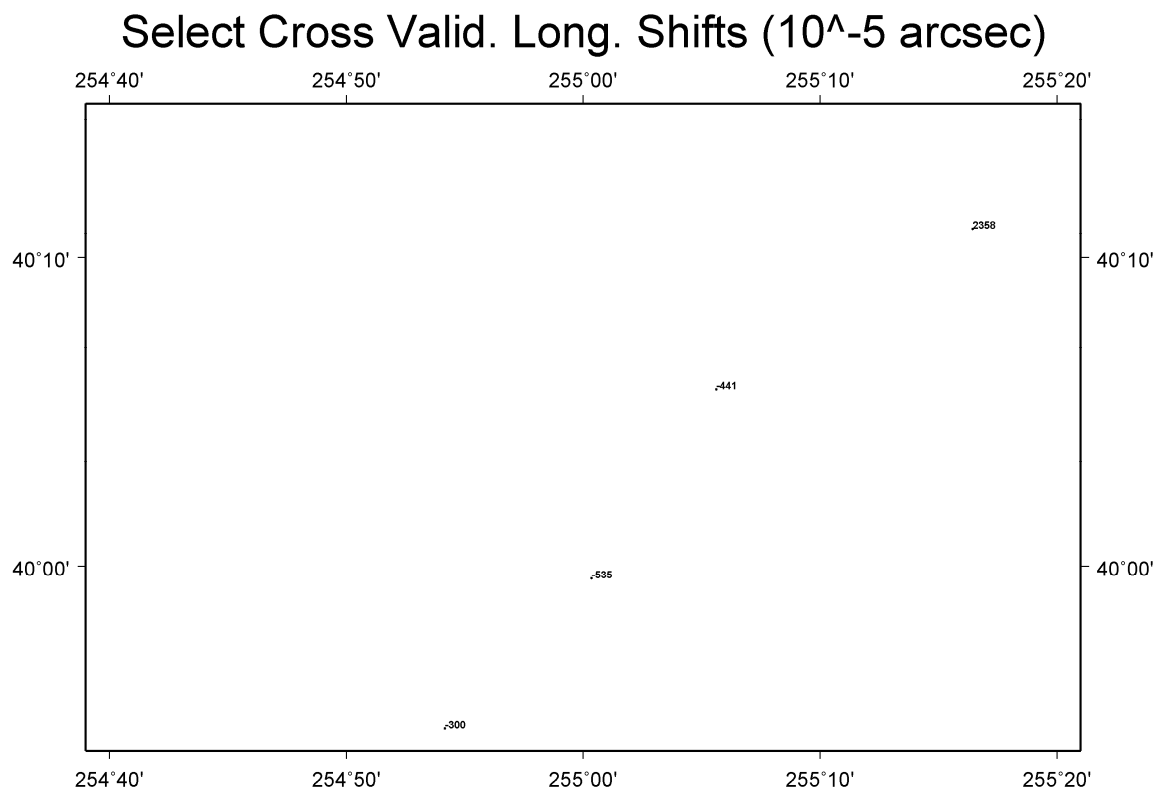


Figure 6.8. Abnormal Cross-Validation Error of Longitude Differences.

The first thing to be noticed in Figure 6.7 is that the nominal values of the cross-validation errors are typically much smaller than the nominal -0.00050 arc second longitude shift. This shows the grid is doing a very good job of predicting the coordinate shift at a point when that point was withheld from the gridding computation.

In addition, the cross-validation errors at the abnormal points do, indeed, look abnormal. They show the systematic characters of the quality value, including the sign of the value. For example, consider LL1465, with an actual longitude shift of +0.02328 arc seconds. The gridded transformation in Figure 6.5 is about -0.00030 arc seconds. So the systematic error for LL1465 is assessed at +0.02358 arc seconds.

Recall that the transformation quality grid is obtained by gridding the cross-validation error, and that abnormalities were dropped by a median filter. However, in the case of clusters of the cross-validation error, the worst-case error is passed to the gridding algorithm. This makes a distinct difference in the locations around KK2067 and LL1465. This is displayed in Figure 6.9.

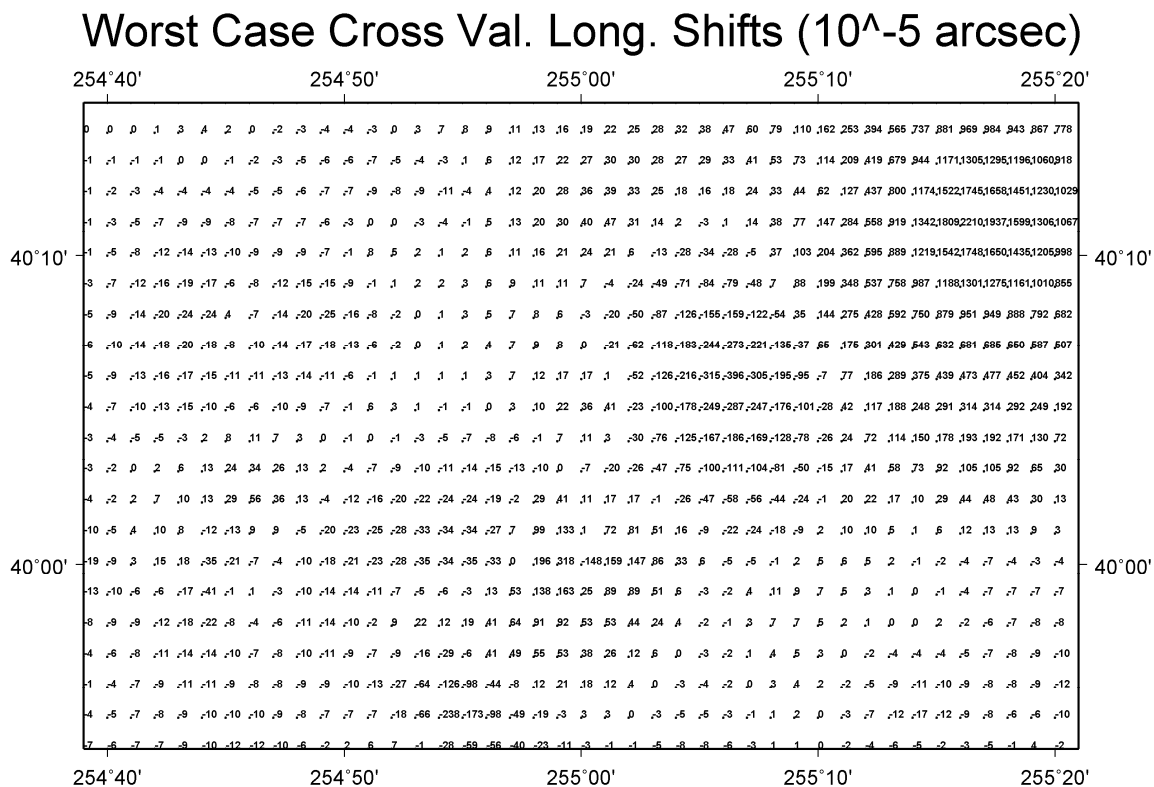


Figure 6.9. Gridded Worst Case Cross-Validation Error of Longitude Differences.

The points KK2064 and LL1240 have isolated abnormal shifts. So, their gridded errors spread out over some distance. The southern point, KK2067, is the worst case in a cluster. But the gridded error quickly decreases as one approaches the normal neighbors. The northeast point, LL1465 is in an isolated cluster. Its worst case cross-validation error passes to the gridding algorithm. And, because the cluster was isolated, the error estimate spreads over a larger distance in the quality grid.

Therefore, we see that the user is warned about the presence of abnormal coordinate shifts in the vicinity of a geospatial project. Large values in the quality grid are created by coordinate shifts that don't agree with neighboring values. And, those large values are reported whether they are isolated, or present in a cluster of normal coordinate shifts.

These quality values indicate potential systematic error in the reported transformation. The error is *not* due to uncertainty in a nearby coordinate shift. The coordinate shifts used to create the GEOCON transformations are, by definition, error-free. Rather, the large quality values indicate uncertainty in how a local project connected into the existing control, and what procedures were followed by the practitioner regarding misfits. The quality values should be considered systematic, and absolute values of the quality numbers should be used to increase the base network accuracy of the pre-transformed coordinates.

As a final part of this example, consider the notification messages. As discussed in the GEOCON Operating Instructions, notification messages may be generated when converting near abnormal coordinate shifts present in a cluster. The pair of abnormal points (KK2067 and LL1465) seen in Figure 6.6 are stored in an information file. Notification messages are issued when an input point has horizontal or vertical error of 5 cm or more and is within about 5 km of an abnormal point in an information file. The notification messages are purely informational, and help indicate the source of a large quality value when a nearby cluster would create some ambiguity in diagnosis.

7. General Quality of the Coordinate Transformations

The quality grids, obtained from 69-fold cross-validation, are highly variable. Even so, it is worthwhile to get a general view of how well the transformation grid is able to predict at withheld points.

The two-tailed percentiles of the distributions of the cross-validation for the conterminous U.S. are collected in Table 7.1. Approximately 68490 points were validated.

Table 7.1 – Percentiles of Cross-Validation Error, CONUS

| Percentile | Latitude (0.00001 arc sec) | Longitude (0.00001 arc sec) | Height (cm) |
|------------|-------------------------------|--------------------------------|----------------|
| 50% | 3.1 | 3.8 | 0.2 |
| 68% | 7.2 | 9.0 | 0.5 |
| 90% | 31.2 | 40.9 | 1.9 |
| 95% | 62.5 | 82.0 | 3.5 |
| 99% | 324.0 | 405.6 | 8.9 |
| 99.9% | 895.7 | 1003.6 | 23.6 |

It is seen that the 95% limits are remarkably good. We have 95% bounds of +/- 1.9 cm in latitude, +/- 2.0 cm in longitude, and +/- 3.5 cm in height. Note that the distribution is not Gaussian. It is very peaked (leptokurtic), with long tails. The 68% bounds are almost 10 times smaller than the 95% bounds. In general, the quality of the

coordinate transformation is remarkably good. In fact, at the 90% level it is comparable to the network accuracy of NAD 83(NSRS2007). Of course, as discussed earlier, the cross-validation error should be treated as systematic. Therefore, the absolute value of the quality numbers should be used to increase the base network accuracy of the pre-transformed coordinates.

The two-tailed percentiles of the distributions of the cross-validation for Alaska are collected in Table 7.2. Approximately 770 points were validated.

Table 7.2 – Percentiles of Cross-Validation Error, Alaska

| Percentile | Latitude (0.00001 arc sec) | Longitude (0.00001 arc sec) | Height (cm) |
|------------|-------------------------------|--------------------------------|----------------|
| 50% | 28.5 | 61.3 | 1.4 |
| 68% | 67.8 | 162.7 | 3.9 |
| 90% | 246.7 | 534.4 | 13.7 |
| 95% | 323.7 | 735.3 | 23.3 |
| 99% | 1091.3 | 1338.0 | 95.6 |
| 99.9% | 1739.5 | 2255.1 | 8602.7 |

It is seen that the 95% limits are much poorer than for the conterminous U. S. We now have 95% bounds of +/- 10.0 cm in latitude, +/- 10.8 cm in longitude, and +/- 23.3 cm in height. Even so, these are sufficient quality to transform many types of geospatial data. Note that because of the small sample size (770), when the limits are established at the 99.9% boundary, height outliers are seen to appear.

The two-tailed percentiles of the distributions of the cross-validation for Puerto Rico/Virgin Islands are collected in Table 7.3. Approximately 145 points were validated.

Table 7.3 – Percentiles of Cross-Validation Error, Puerto Rico/Virgin Islands

| Percentile | Latitude (0.00001 arc sec) | Longitude (0.00001 arc sec) | Height (cm) |
|------------|-------------------------------|--------------------------------|----------------|
| 50% | 5.3 | 7.9 | 0.4 |
| 68% | 8.7 | 15.6 | 0.7 |
| 90% | 37.1 | 82.3 | 2.3 |
| 95% | 64.3 | 169.0 | 3.1 |
| 99% | 112.0 | 400.6 | 5.2 |
| 99.9% | 120.3 | 434.7 | 9.9 |

Here the 95% limits fall between those of the conterminous U. S. and Alaska. The 95% bounds are +/- 2.0 cm in latitude, +/- 5.2 cm in longitude, and 3.1 cm in height. Note that the relatively small values at the 99% and 99.5% limits can not be given much interpretation, since the sample size is so small (145).

Note that the limits reported in Tables 7.1, 7.2, and 7.3 are general statistics. The cross-validation error is highly variable. For this reason, one should refer to the *94* transformation quality records issued by GEOCON for specific projects.

8. Hawaii

There is no official NSRS2007 coordinate for Hawaii. However, a control point, KOKEE, (PID = TT3487) was inadvertently published in the NGS database as NSRS2007. This NSRS2007 position was subsequently removed from the NGS database in 2012. Hence, there are no GEOCON coordinate transformations for Hawaii.

9. Images of the Transformation and Quality Grids

At this point it is useful to provide some images of the coordinate transformations from the NAD 83(HARN) to the NAD 83(NSRS2007), and the associated transformation quality grids. In this guide, only images for the conterminous U. S. are provided. However, many more images are available in the Electronic Support Material (ESM) of the GEOCON Technical Report. Figure 9.1 displays horizontal coordinate differences.

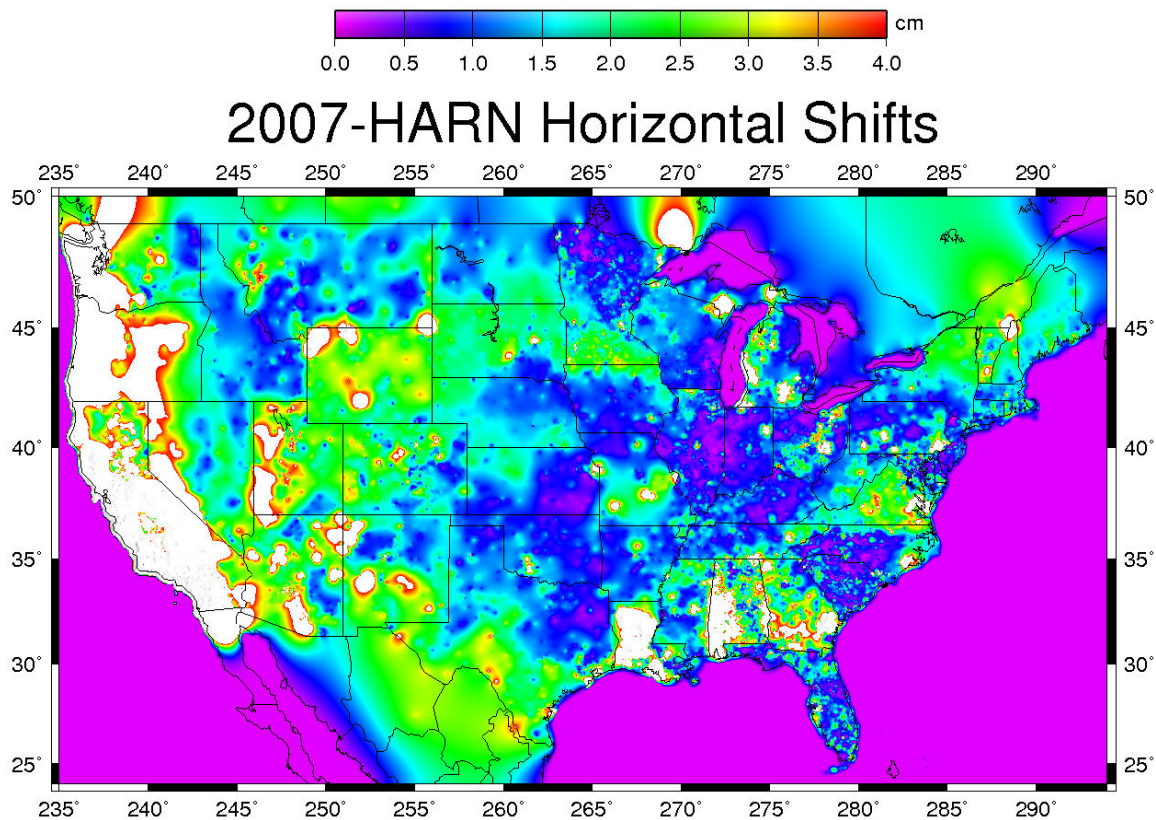


Figure 9.1. Horizontal Coordinate Differences.

The white areas in Figure 9.1 are where the horizontal shift exceeds the color scale of 4 cm. The most obvious shifts are along the West Coast of the U. S. This reflects the crustal motion of the Pacific plate between the years of 2002 and 2007 (the epochs of the HARN and the NSRS2007). Values in Canada and Mexico are to be disregarded. The coordinate shifts in the oceans and Great Lakes were set to near-zero to control the gridding.

Also notable in Figure 9.1 are large shifts in Louisiana and portions of Alabama and Georgia. It is also seen that the color transitions sometimes conform to state boundaries. This is because the HARN adjustments were performed on groupings of one or more states (Milbert and Milbert, 1994).

And, it must be pointed out that the horizontal shifts in most of the country exceed the typical horizontal network accuracy of 1 cm (Milbert 2008). This demonstrates that the NSRS2007 National Readjustment was necessary to obtain those excellent network accuracies.

Next, Figure 9.2 shows the ellipsoidal height coordinate differences. White areas are where the vertical shift is greater than the color scale, and black areas are where the shift is smaller than the color scale. Of note are vertical shifts in portions of California, southern Minnesota, and Alabama.

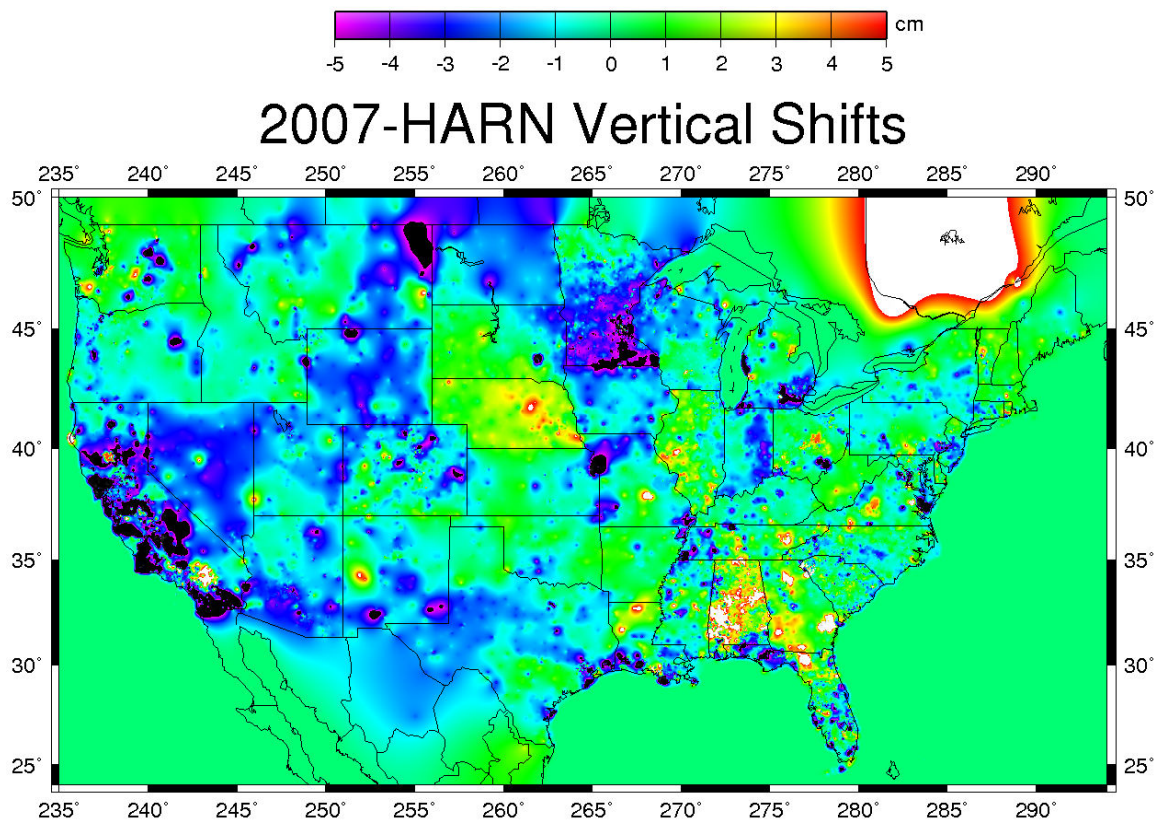


Figure 9.2. Ellipsoidal Height Coordinate Differences.

Figures 9.3, 9.4, and 9.5 plot the worst case cross-validation errors in latitude, longitude, and ellipsoidal height. These portray the transformation quality grids.

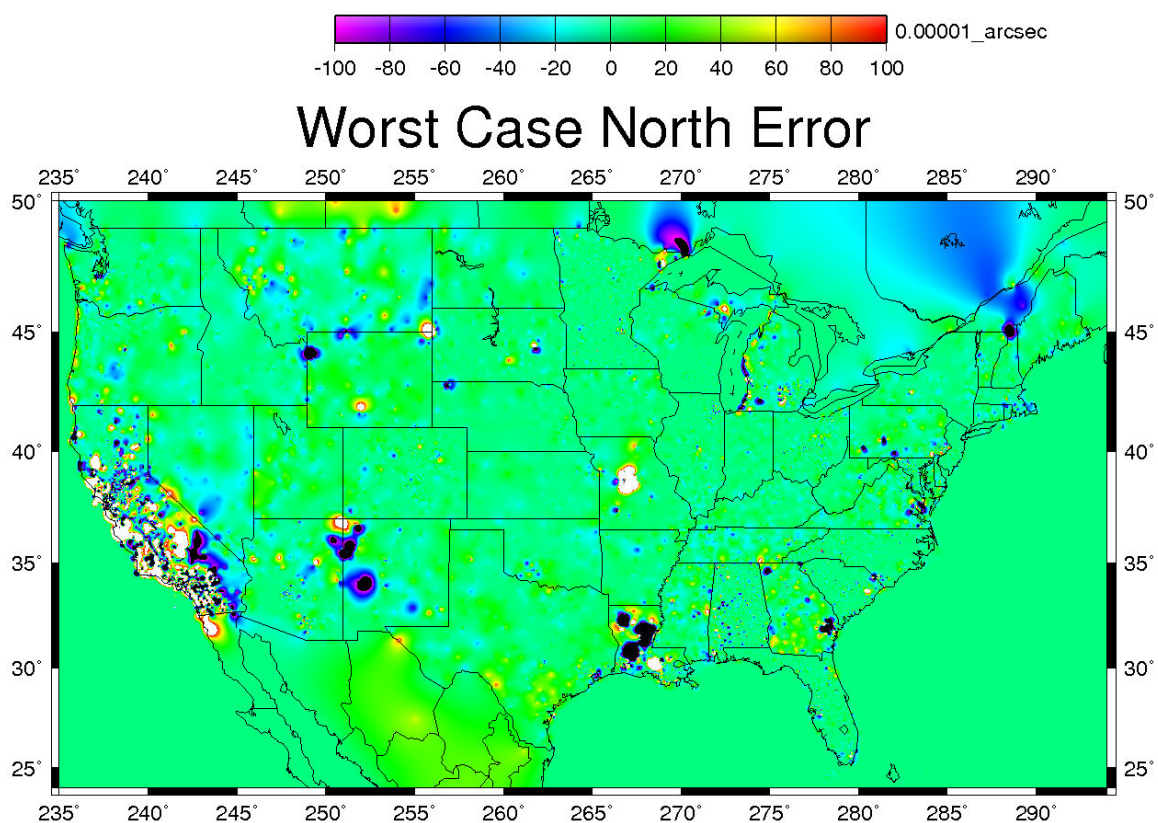


Figure 9.3. Worst Case Cross Validation Error, Latitude.

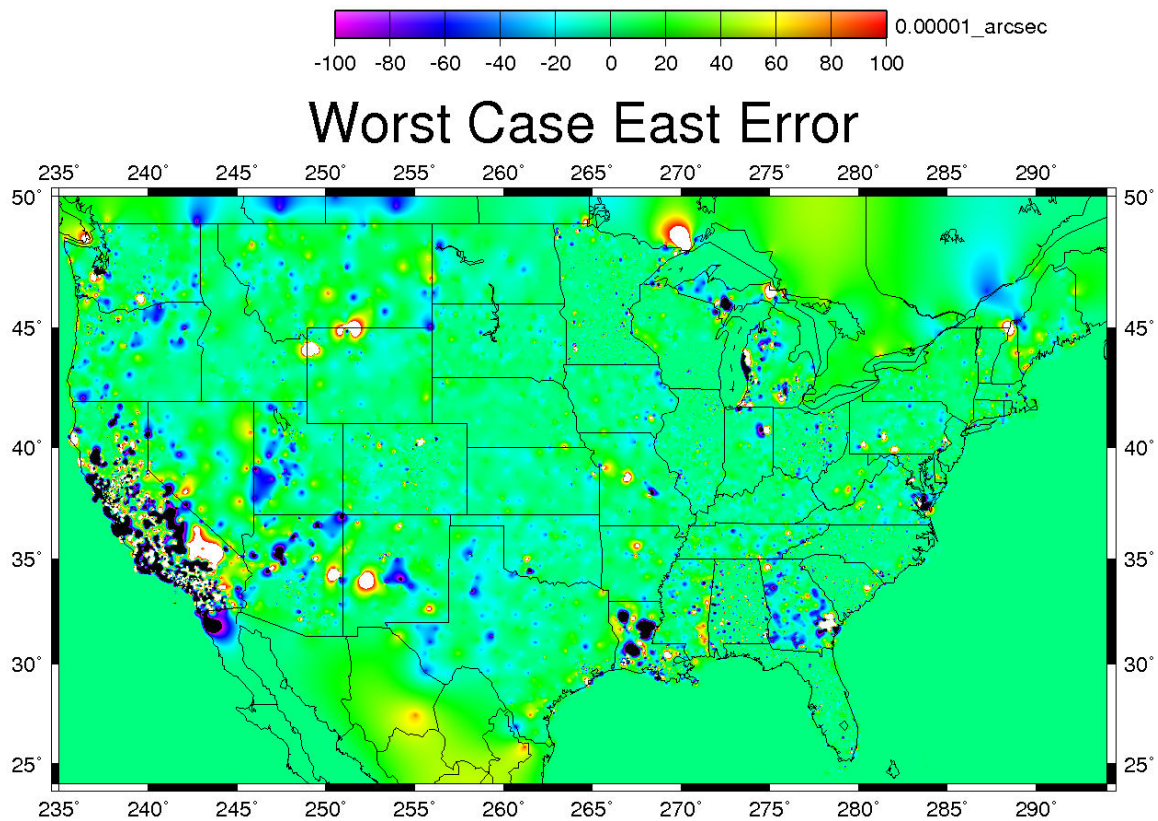


Figure 9.4. Worst Case Cross-Validation Error, Longitude.

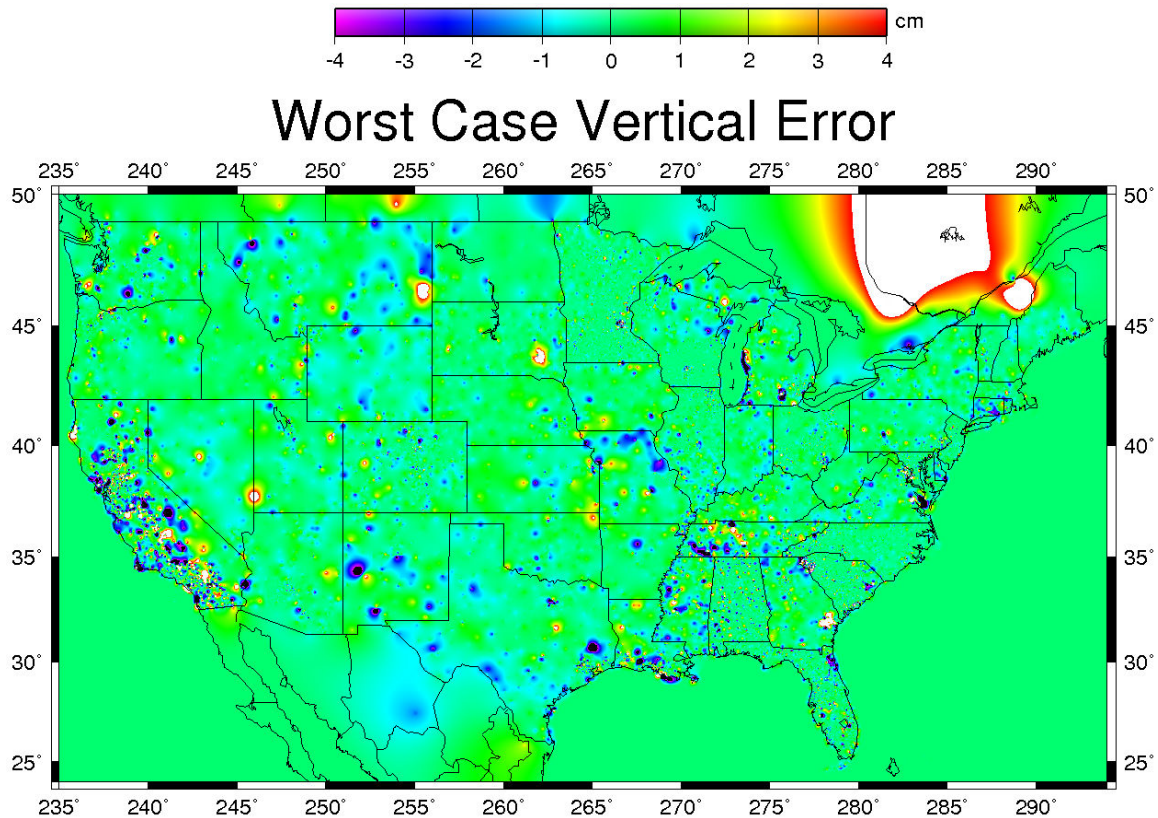


Figure 9.5. Worst Case Cross-Validation Error, Ellipsoidal Height.

The error grids should be inspected in conjunction with the percentiles of Table 7.1. It is seen that the coordinate shifts can be predicted quite well. The abnormal cases are quite sporadic. California and Louisiana have the most troublesome horizontal coordinate shifts. However, the vertical shifts in Louisiana are seen to be modeled somewhat better in GEOCON.

10. Case Study 1

For the first case study, consider a GPS relative carrier phase survey in Arizona: Project GPS2828. It was observed in February 2006, and was processed in December 2010. The survey extends about 1 degree in latitude and 1.7 degrees in longitude. This survey includes 45 points, including 2 CORS. Eight passive marks were existing control points.

This project provides a means of testing the predictive capability of the GEOCON coordinate transformation. While the raw data existed in 2006, it was not included in the NSRS2007 National Readjustment. The GPS vectors were not generated until the end of 2010. The test will be to see how well HARN coordinates can be transformed from starting NSRS2007 coordinates.

First, a constrained adjustment by means of program ADJUST (Milbert and Kass, 1987) was computed for the NSRS2007 coordinates. Eight passive control points were

held fixed to the NGS database NSRS2007 values. The resulting set of 45 coordinates in NSRS2007 is denoted `bbook2007.txt`.

Then, the `*80/*86*` NSRS2007 coordinate records were extracted from `bbook2007.txt`, and transformed from NSRS2007 into HARN by program GEOCON. These transformed coordinates are designated `8086-harn.txt`. No points were clipped, and no notifications were issued. The quality values were all sub-centimeter, and were typically only a few millimeters.

Then, a “truth” coordinate set is generated by another constrained adjustment computed for the HARN coordinate. The same set of 8 control points were held fixed to the database HARN values. This control set in the HARN is named `bbookharn.txt`.

Finally, a simple program, `harnstat`, was written to compare the HARN coordinates between the transformed set and the control set obtained by the HARN constrained adjustment. The output of `harnstat` is presented in Appendix A.2. The sense of the signs is: transformed – adjusted. Longitudes are taken as positive East. All units are millimeters. The results, including the Root Mean Square (RMS) about zero and about the mean, are summarized in Table 10.1.

Table 10.1 – Summary Statistics for the Transformation of GPS2828

| Percentile | Latitude (mm) | Longitude (mm) | Height (mm) |
|------------|------------------|-------------------|----------------|
| Average | -0.3 | -0.6 | 1.1 |
| RMS, zero | 1.5 | 4.2 | 2.6 |
| RMS, mean | 1.5 | 4.1 | 2.4 |

It is seen that GEOCON did a very good job of transforming the NSRS2007 coordinates into the HARN. The RMS about the mean is roughly comparable to the variety of quality values issued by GEOCON.

Of course, in this first example, the GPS survey happens to tie into points that behave very much like their neighbors. None of the 8 fixed points can be considered as having abnormal coordinate shifts between HARN and NSRS2007.

11. Case Study 2: Abnormal Points

The second case study will consist of 3 parts. However, all 3 parts will use an identical set of GPS vectors. Consider a synthetic GPS project in Northeast Colorado shown in Figure 11.1.

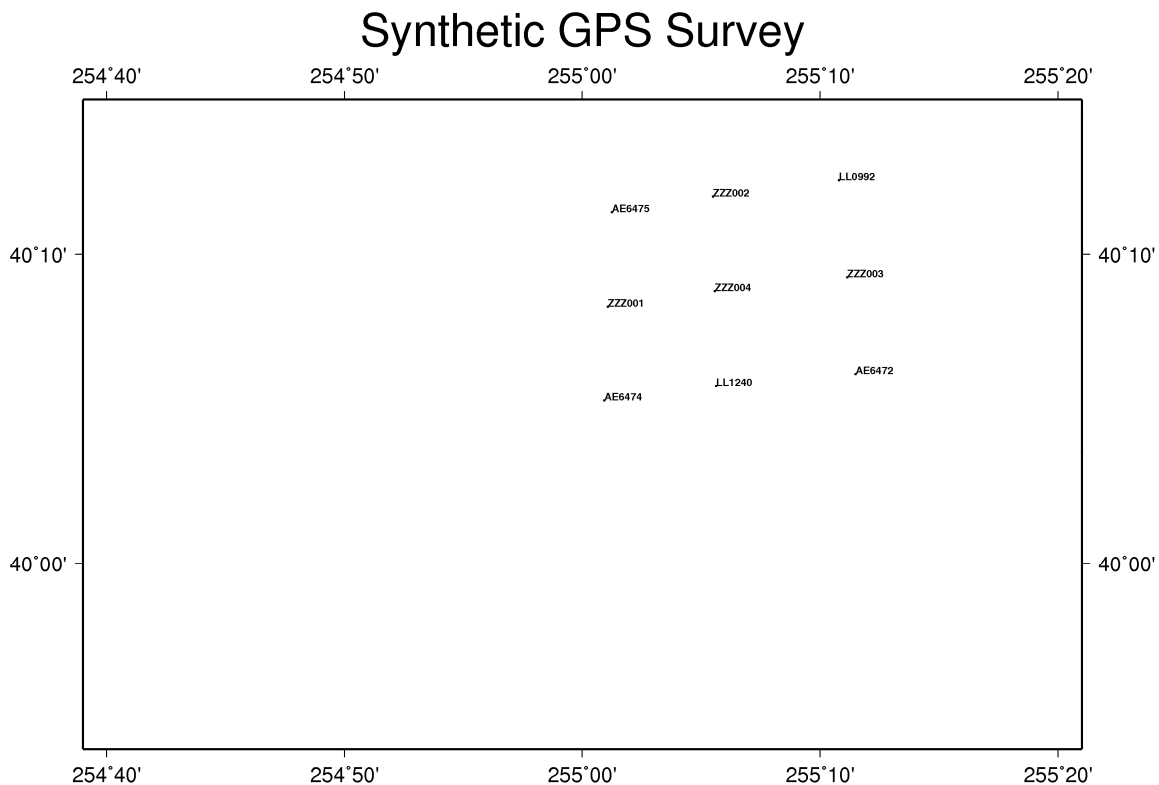


Figure 11.1. A Simulated GPS Survey.

This example represents densification with 4 new points labeled with the prefix “ZZZ”. The remaining 5 points are the same control points found in Section 6. The objective is to get a set of HARN coordinates for the new points from NSRS2007 coordinate values using GEOCON. Recall that the coordinate shift difference at LL1240 was abnormal (about 12 cm in longitude). It is important to note that the new points’ survey connections to the established control are excellent.

The synthetic GPS data consists of 12 vectors, each in a separate session, with no correlations between components. Each vector connects northward or eastward to an adjacent point. This creates figures of quadrilaterals with 2 vectors in each of the 3 rows and 3 columns. The vector components were created from preliminary coordinates based on NSRS2007. Gaussian random noise of 0.5 cm standard deviation was computed by a Box-Mueller method (Forsythe *et al.*, 1977, pg. 247), and added to each synthetic component.

The first part of this case study begins with a constrained adjustment of AE6474, LL1240, AE6472, AE6743, and LL0992 in NSRS2007. The resulting set of 9 coordinates in NSRS2007 is denoted `bbook2007.txt`.

As in the first case study, the *80*/*86* NSRS2007 coordinate records were extracted from `bbook2007.txt`, and transformed from NSRS2007 into HARN by program GEOCON. These transformed coordinates are designated `8086-harn.txt`. No points were clipped, and no notifications were issued. But, the quality values ranged up to 8 cm. This is due to the influence of the longitude shift at LL1240.

The control coordinate set is generated by another constrained adjustment computed for the HARN. The same set of 5 control points were held fixed to the database HARN values. This control set in the HARN is named `bbookharn.txt`.

As with the first case study, the transformed coordinates are compared against the HARN-adjusted coordinates. Results in millimeters are displayed in Table 11.1

Table 11.1 – Part 1, 5 Fixed Control Points (units of mm)

| | | | | | | | |
|--|------|------|-------|-----------------|------------------------|-------|-------|
| program harnstat -- 2012jan18 | | | | | | | |
| input 80/86 file (input) : 8086-harn.txt | | | | | | | |
| input bbook file (input) : bbookharn.txt | | | | | | | |
| 9 points loaded | | | | | | | |
| | | | | test comparison | transformation quality | | |
| AE6474 | 0001 | 0.0 | 1.9 | 0.0 | 4.6 | 9.5 | -9.9 |
| LL1240 | 0002 | 0.6 | -16.7 | 2.0 | 3.8 | -89.7 | 12.8 |
| AE6472 | 0003 | 0.3 | 3.1 | -1.0 | 3.1 | 32.2 | -10.3 |
| ZZZ001 | 0004 | 0.3 | 2.8 | 0.0 | 2.9 | -4.4 | -3.5 |
| ZZZ004 | 0005 | 0.0 | 5.3 | -3.0 | 3.7 | -22.8 | 1.1 |
| ZZZ003 | 0006 | 1.9 | -2.8 | -2.0 | 11.2 | 89.9 | -24.7 |
| AE6475 | 0007 | 0.0 | 1.9 | 0.0 | 0.4 | 11.1 | 1.3 |
| ZZZ002 | 0008 | 0.3 | 0.0 | -5.0 | 2.4 | 3.4 | -0.6 |
| LL0992 | 0009 | 0.3 | 0.9 | -1.0 | 4.6 | 23.0 | -10.7 |
| 9 points processed | | | | | | | |
| ave: | 0.4 | -0.4 | -1.1 | | | | |
| rms0 | 0.7 | 6.1 | 2.2 | | | | |
| rms: | 0.5 | 6.1 | 1.9 | | | | |

GEOCON has done a surprisingly good job in transforming the coordinates from NSRS2007 into the HARN despite the abnormal shift at LL1240 (serial number 0002). In fact, the -16.7 mm longitude difference between the two coordinate sets at LL1240 shows GEOCON has some difficulty in exactly modeling the full amount of the shift at that point. The reason for these excellent results is that the synthetic survey tied into all the surrounding control. This is exactly the scenario that is modeled by the spine fitting method used to develop the GEOCON grids.

In the second part, we keep the same GPS survey. But suppose that the survey was unable to connect into LL1240. Perhaps the point couldn't be recovered. Perhaps it was destroyed when the synthetic survey was performed. In any case, at that location a new point is established, named ZZZ005 (serial number 0002).

The same procedures are followed. An adjustment is performed with 4 fixed NSRS2007 control points. The 9 coordinate records are transformed from NSRS2007 to HARN with GEOCON. To compare, the same 4 fixed point adjustment is done in the HARN. The transformed coordinates are compared to the adjusted coordinates and given in millimeters in Table 11.2.

Table 11.2 – Part 2, 4 Fixed Control Points (units of mm)

| | | | | | | | |
|--|------|-----|-----|-----------------|------------------------|-----|------|
| program harnstat -- 2012jan18 | | | | | | | |
| input 80/86 file (input) : 8086-harn.txt | | | | | | | |
| input bbook file (input) : bbookharn.txt | | | | | | | |
| 9 points loaded | | | | | | | |
| | | | | test comparison | transformation quality | | |
| AE6474 | 0001 | 0.0 | 1.9 | 0.0 | 4.6 | 9.5 | -9.9 |

| | | | | | | | |
|--------------------|------|------|-------|-------|------|-------|-------|
| ZZZ005 | 0002 | -3.4 | 119.2 | -12.0 | 3.8 | -89.7 | 12.8 |
| AE6472 | 0003 | 0.3 | 3.1 | -1.0 | 3.1 | 32.2 | -10.3 |
| ZZZ001 | 0004 | -0.3 | 17.9 | -2.0 | 2.9 | -4.4 | -3.5 |
| ZZZ004 | 0005 | -1.2 | 50.7 | -8.0 | 3.7 | -22.8 | 1.1 |
| ZZZ003 | 0006 | 1.5 | 12.4 | -3.0 | 11.2 | 89.9 | -24.7 |
| AE6475 | 0007 | 0.0 | 1.9 | 0.0 | 0.4 | 11.1 | 1.3 |
| ZZZ002 | 0008 | 0.0 | 15.1 | -7.0 | 2.4 | 3.4 | -0.6 |
| LL0992 | 0009 | 0.3 | 0.9 | -1.0 | 4.6 | 23.0 | -10.7 |
| 9 points processed | | | | | | | |
| ave: | -0.3 | 24.8 | -3.8 | | | | |
| rms0 | 1.3 | 44.1 | 5.5 | | | | |
| rms: | 1.3 | 36.5 | 4.0 | | | | |

GEOCON applied a very large longitude shift where ZZZ005 (serial number 0002) is located. And it also applied about half that amount at ZZZ004 (serial number 0005). These happen to be erroneous shifts. This is because in this second part, the synthetic GPS survey was connected solely to points that had common coordinate shifts. And, GEOCON is modeling the longitude abnormality at LL1240.

In the third part of the case study, we keep the same GPS survey. But, suppose that the survey was connected to only one point, LL1240. In this case we have new points ZZZ005, ZZZ006, ZZZ007, and ZZZ008 replacing the old control at the same locations. This is a total of 8 new points, and one old point.

The same adjustment and transformation procedures are followed as before, however with just one fixed point. The transformed coordinates are compared to the adjusted coordinates and given in millimeters in Table 11.3.

Table 11.3 – Part 3, 1 Fixed Control Point (units of mm)

| | | | | | | | |
|--|------|--------|--------|-----------------|------------------------|-------|-------|
| program harnstat -- 2012jan18 | | | | | | | |
| input 80/86 file (input) : 8086-harn.txt | | | | | | | |
| input bbook file (input) : bbookharn.txt | | | | | | | |
| 9 points loaded | | | | | | | |
| | | | | test comparison | transformation quality | | |
| ZZZ005 | 0001 | 4.6 | -132.8 | 7.0 | 4.6 | 9.5 | -9.9 |
| LL1240 | 0002 | 0.6 | -16.7 | 2.0 | 3.8 | -89.7 | 12.8 |
| ZZZ006 | 0003 | 2.8 | -133.1 | 26.0 | 3.1 | 32.2 | -10.3 |
| ZZZ001 | 0004 | 4.6 | -119.2 | 0.0 | 2.9 | -4.4 | -3.5 |
| ZZZ004 | 0005 | 3.1 | -86.2 | 0.0 | 3.7 | -22.8 | 1.1 |
| ZZZ003 | 0006 | 5.6 | -124.5 | 8.0 | 11.2 | 89.9 | -24.7 |
| ZZZ007 | 0007 | 5.3 | -137.4 | -8.0 | 0.4 | 11.1 | 1.3 |
| ZZZ002 | 0008 | 4.9 | -122.9 | -6.0 | 2.4 | 3.4 | -0.6 |
| ZZZ008 | 0009 | 5.6 | -136.5 | 2.0 | 4.6 | 23.0 | -10.7 |
| 9 points processed | | | | | | | |
| ave: | 4.1 | -112.2 | 3.4 | | | | |
| rms0 | 4.4 | 118.0 | 10.0 | | | | |
| rms: | 1.6 | 36.8 | 9.4 | | | | |

This third part of this case study created a HARN coordinate set that was completely anchored to LL1240. It was only at LL1240 that GEOCON was able to issue a coordinate shift consistent with that particular adjustment.

The longitude transformation quality values in this second case study have an estimated error of -9 cm at LL1240, and -2.3 cm at ZZX004. These errors are much too

large for part one of this case study. However, we had ideal connections to both normal and abnormal network control points in part one. These same errors are a little small, but about right for part two. In part two, the survey was only connected to control points with normal longitude shifts. And, these same quality values fail to show the reverse distribution of error seen in part three. Of course, part three is a pathological situation; where the survey is connected solely to an isolated, abnormal control point, and all the existing control in the area was ignored.

As described in Section 6, the cross-validation errors estimate how well one can predict a value at a withheld point. Hence, they can quantify abnormality of a coordinate difference. This quality model is most appropriate when one only connects to normal points. The quality values overestimate when one connects into both normal and abnormal points. But, cross-validation was deliberately selected to provide conservative estimates of error. This is similar to the decision to choose the worst case cross-validation error in the case of a point cluster: provide maximal warning to the user. In the end, the quality values do not reflect survey accuracy. Rather, they indicate our lack of knowledge about how the input coordinates are connected to the national network.

These case studies provide a deeper explanation of why the National Geodetic Survey considers actual recomputation of geospatial data, and not coordinate transformations, as “best practice”. The coordinate transformation is, at its heart, only a model of actual geospatial measurement and processing.

In closing this section, it should be noted that the results would have been the same if the densification consisted of a grid of 400 new, interconnected points instead of 4. And, photogrammetric mosaics, or other interconnected geospatial positioning data, would behave similarly to synthetic GPS vectors.

12. Case Study 3: Clusters

The third case study illustrates behavior when control points are clustered. This study also consists of 3 parts. The general approach is similar to the second case study. The study involves a synthetic GPS project somewhat further to the northeast than seen in the second case study. This example also presents a densification survey with 3 new points labeled with “ZZZ”. The arrangement of points is depicted in Figure 12.1.

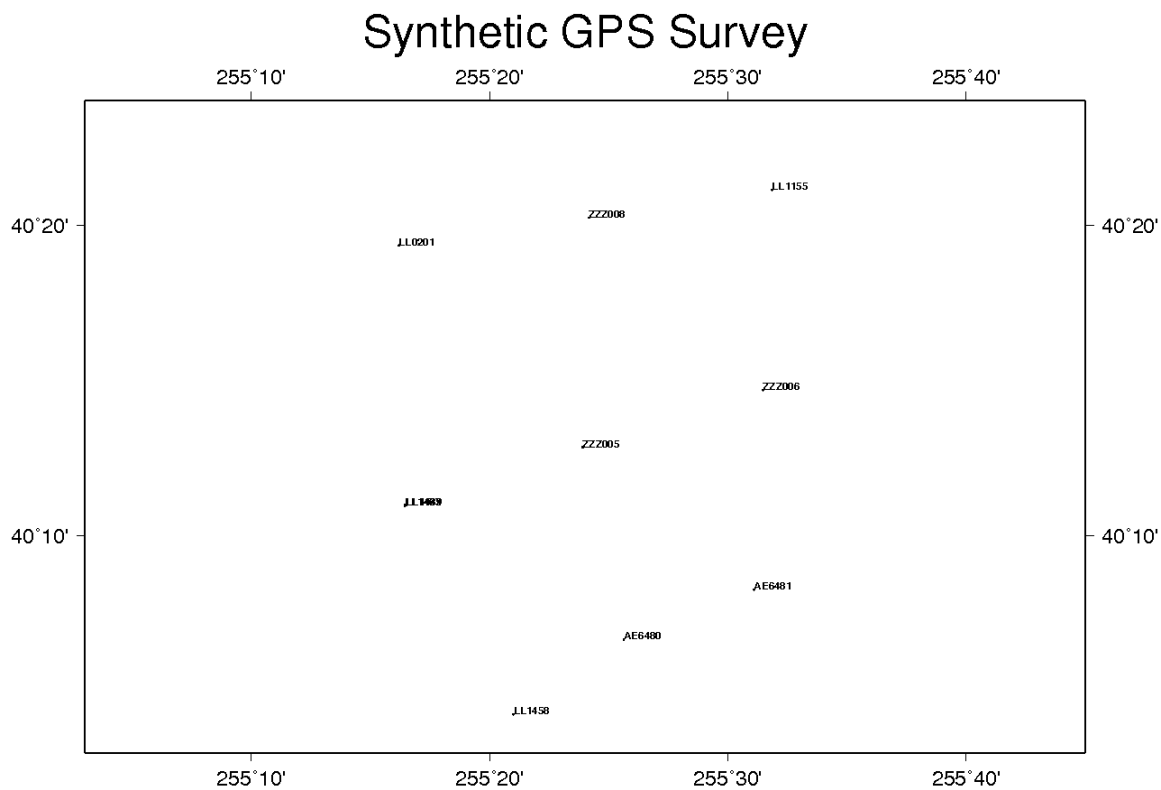


Figure 12.1. Another Simulated GPS Survey.

The West-central location in the array is a cluster with 7 points. For the purposes of this case study, the cluster is thinned down to 3 points: LL1439, LL1465, and LL1477. LL1439 is a point that generally agrees with its neighbors in the network. LL1465 and LL1477, however, have abnormal coordinate shifts.

The synthetic GPS data consists of 12 vectors with the same general arrangement as in the second case study. However, due to the cluster at the West-Central location, the choice of control point changes for each part of this three-part study. Therefore the synthetic GPS data are regenerated anew for each part.

In the first part of the study, the synthetic survey ties to LL1439, a normal point. A set of synthetic data was generated using the NSRS2007 coordinates and adding Gaussian random noise. A constrained adjustment was performed with LL1458, AE6480, AE6481, LL1155, LL0201, and LL1439 fixed in NSRS2007. The resulting coordinates were transformed with GEOCON into the HARN. As in the second case study, a separate adjustment of the synthetic GPS data was computed in the HARN. The transformed coordinates are compared to the adjusted coordinates and given in millimeters in Table 12.1.

Table 12.1 – Part 1, All Ties to Normal Points (units of mm)

| | | |
|-------------------------------|--------------------|------------------------|
| program harnstat -- 2012jan18 | | |
| input | 80/86 file (input) | : 8086-harn.txt |
| input | bbook file (input) | : bbookharn.txt |
| | 9 points loaded | |
| | test comparison | transformation quality |

| | | | | | | | |
|--------------------|------|------|------|-------|------|-------|--------|
| LL1458 | 0001 | 0.3 | 0.3 | 0.0 | 5.3 | 7.2 | -10.7 |
| AE6480 | 0002 | 0.0 | -0.3 | 0.0 | 1.8 | 5.6 | -2.8 |
| AE6481 | 0003 | 0.0 | 0.0 | 0.0 | 1.4 | 0.9 | -6.2 |
| LL1439 | 0004 | 3.4 | 0.6 | 2.0 | 51.2 | 510.3 | -116.8 |
| ZZZ005 | 0005 | 1.9 | -0.9 | 0.0 | 12.5 | 115.1 | -27.3 |
| ZZZ006 | 0006 | 0.9 | -0.3 | 2.0 | 3.7 | 26.1 | -8.6 |
| LL0201 | 0007 | 0.3 | 0.0 | 0.0 | 4.5 | 4.7 | 2.6 |
| ZZZ008 | 0008 | 7.4 | -4.6 | -10.0 | 1.5 | 30.1 | -3.9 |
| LL1155 | 0009 | 0.3 | -0.3 | 0.0 | 3.8 | -2.8 | 0.5 |
| 9 points processed | | | | | | | |
| ave: | 1.6 | -0.6 | -0.7 | | | | |
| rms0 | 2.8 | 1.6 | 3.5 | | | | |
| rms: | 2.3 | 1.5 | 3.4 | | | | |

It is seen that the transformed set of HARN coordinates have excellent agreement with the coordinates generated by the readjustment of the synthetic data in the HARN coordinate. However, very large transformation quality values are issued by GEOCON. In addition, GEOCON issues a pair of notification messages:

```
*80*0004LL1439                                40105728646N104433127706W150124
Note - poor quality due to LL1465 - unmodeled hztl. error:      72.8 cm.
*86*0004 1501245                                000 1501267
Note - poor quality due to LL1477 - unmodeled vert. error:     -12.6 cm.
```

Both messages refer to station ID 0004, which is LL1439. That point is completely benign and had a good transformation. However, it is known that that NGS database included points LL1465 and LL1477, and they both had abnormal coordinate shifts. The GEOCON transformation grids do not model the behavior of those 2 points. They were in a cluster of 7 points, and were dropped in the gridding process. But, the transformation quality grid is based on the worst case behavior of the cross-validation error. The large values in the transformation quality are a warning survey connections to the troublesome points will lead to coordinate conversion problems. The notifications explain that LL1439 is near those troublesome points.

It happens that the survey in the first part only connected to control with normal coordinate shifts. Hence, the transformation warnings seem much too large. Consider a similar situation, where the synthetic survey does tie to an abnormal point, LL1465.

In the second part, point, LL1465 replaces LL1439. A new set of synthetic data was generated using the NSRS2007 coordinates and adding Gaussian random noise. A constrained adjustment was performed with LL1458, AE6480, AE6481, LL1155, LL0201, and LL1465 fixed in NSRS2007. The resulting coordinates were transformed with GEOCON into the HARN. As in the first part, a separate adjustment of the synthetic GPS data was computed in the HARN. The transformed coordinates are compared to the adjusted coordinates and given in millimeters in Table 12.2.

Table 12.2 – Part 2, Tie to Abnormal Point, LL1465 (units of mm)

| | | | | | | | |
|--|------|-----|-----|-----------------|------------------------|-----|-------|
| program harnstat -- 2012jan18 | | | | | | | |
| input 80/86 file (input) : 8086-harn.txt | | | | | | | |
| input bbook file (input) : bbookharn.txt | | | | | | | |
| 9 points loaded | | | | | | | |
| | | | | test comparison | transformation quality | | |
| LL1458 | 0001 | 0.3 | 0.3 | 0.0 | 5.3 | 7.2 | -10.7 |

| | | | | | | | |
|--------------------|------|-------|-------|-------|------|-------|--------|
| AE6480 | 0002 | 0.0 | -0.3 | 0.0 | 1.8 | 5.6 | -2.8 |
| AE6481 | 0003 | 0.0 | 0.0 | 0.0 | 1.4 | 0.9 | -6.2 |
| LL1465 | 0004 | 55.6 | 728.3 | -28.0 | 51.4 | 512.2 | -117.2 |
| ZZZ005 | 0005 | 17.0 | 217.8 | -8.0 | 12.5 | 115.1 | -27.3 |
| ZZZ006 | 0006 | 5.9 | 72.6 | -1.0 | 3.7 | 26.1 | -8.6 |
| LL0201 | 0007 | 0.3 | 0.0 | 0.0 | 4.5 | 4.7 | 2.6 |
| ZZZ008 | 0008 | 12.7 | 68.3 | -13.0 | 1.5 | 30.1 | -3.9 |
| LL1155 | 0009 | 0.3 | -0.3 | 0.0 | 3.8 | -2.8 | 0.5 |
| 9 points processed | | | | | | | |
| ave: | 10.2 | 120.7 | -5.6 | | | | |
| rms0 | 19.9 | 255.6 | 10.6 | | | | |
| rms: | 17.1 | 225.2 | 9.1 | | | | |

Inspection of Table 12.2 shows that the transformation quality errors issued in parts one and two of this case study were not too large. In fact, the quality values underestimate the actual error between the transformed and the adjusted coordinate sets. Recall, the same GEOCON transformation grids were used in parts one and two. In the first part, the transformation performed excellently. In the second part, the transformation did not. The difference is due to the way the local data tied into the network. NGS can not know how such projects connect into the national network. But, NGS can warn the user that potential problems might exist.

In the third part, point, LL1477 replaces LL1439 and/or LL1465. New synthetic data was generated as before. A constrained adjustment was performed with LL1458, AE6480, AE6481, LL1155, LL0201, and LL1477 fixed in the NSRS2007 coordinate. The remaining procedures follow parts one and two. The transformed coordinates are compared to the adjusted and given in millimeters in Table 12.3.

Table 12.3 – Part 3, Tie to Abnormal Point, LL1477 (units of mm)

| | | | | | | | |
|--|------|-----------------|------|--------|------------------------|-------|--------|
| program harnstat -- 2012jan18 | | | | | | | |
| input 80/86 file (input) : 8086-harn.txt | | | | | | | |
| input bbook file (input) : bbookharn.txt | | | | | | | |
| 9 points loaded | | | | | | | |
| | | test comparison | | | transformation quality | | |
| LL1458 | 0001 | 0.3 | 0.3 | 0.0 | 5.3 | 7.2 | -10.7 |
| AE6480 | 0002 | 0.0 | -0.3 | 0.0 | 1.8 | 5.6 | -2.8 |
| AE6481 | 0003 | 0.0 | 0.0 | 0.0 | 1.4 | 0.9 | -6.2 |
| LL1477 | 0004 | -0.9 | -4.3 | -126.0 | 51.4 | 512.6 | -117.3 |
| ZZZ005 | 0005 | 0.3 | -2.2 | -38.0 | 12.5 | 115.1 | -27.3 |
| ZZZ006 | 0006 | 0.3 | -0.6 | -11.0 | 3.7 | 26.1 | -8.6 |
| LL0201 | 0007 | 0.3 | 0.0 | 0.0 | 4.5 | 4.7 | 2.6 |
| ZZZ008 | 0008 | 7.1 | -4.9 | -22.0 | 1.5 | 30.1 | -3.9 |
| LL1155 | 0009 | 0.3 | -0.3 | 0.0 | 3.8 | -2.8 | 0.5 |
| 9 points processed | | | | | | | |
| ave: | 1.6 | -0.6 | -0.7 | | | | |
| rms0 | 2.8 | 1.6 | 3.5 | | | | |
| rms: | 2.3 | 1.5 | 3.4 | | | | |

The 7 point cluster that is a foundation for this study contains 2 different anomalous points: LL1465 and LL1477. The first is largely anomalous in latitude and the second is anomalous in height coordinate shift. Table 12.3 illustrates what occurs when the survey ties to the point with the anomalous height shift. The transformation error is distributed very much the same as seen in part two, only in the height component. The transformation quality errors are reflecting both anomalous points.

13. Summary User Guidance

If at all possible, consider reprocessing your geospatial data using control points in the coordinate set you wish to realize. In the case of geodetic survey data, this would entail performing a least squares adjustment with new coordinates as constraints. Notwithstanding the high quality of the GEOCON transformation, the National Geodetic Survey considers actual readjustment of survey measurements, and not coordinate transformations, as “best practice”.

It is understood that for many cases it will not be economical, or may be impossible to work with the original geospatial measurements. Further, the network accuracy of the geospatial data set may be a much larger number than the expected quality value of the coordinate transformation. GEOCON provides an attractive solution.

The quality values reported by GEOCON in the *94* records are systematic, and their absolute values should be added to the base network accuracy of the pre-transformed coordinates. If your transformed accuracies meet your needs, then congratulations, you are done.

However, there may be some spots where the transformation does not have the desired quality. These are caused by proximity to abnormal coordinate differences. The abnormal point or points may be isolated or in clusters. Either way, the abnormality will be expressed in the quality grids. Short of reprocessing the raw data, one is faced with researching how the raw data was connected, directly or indirectly, to the network.

In the case of isolated anomalous points, the transformation grid is modeling the general behavior of a data set connected to every point in the region including the anomalous point. If this is the case for the original raw data, the coordinate transformation will be better than the reported quality. On the other hand, if the raw data connected solely to the normal points, or solely to the isolated anomalous point, then the transformation should be applied manually.

In the case of an anomalous point in a cluster, the transformation grid is modeling the connection to the normal points, including the median normal point in the cluster. If this is the case for the original raw data, the coordinate transformation will be better than the reported quality. However, if the raw data connected to the anomalous point and other normal points, then one must estimate how the anomalous coordinate shift will be distributed between the anomalous point and its neighbors. And, if the raw data connected solely to the anomalous point in the cluster, then the transformation based on the anomalous point should be applied manually.

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About the Author

Dr. Dennis Milbert received his Bachelors degree in Physics from the University of Colorado, and his M.S. and Ph.D. degrees in Geodetic Science from The Ohio State University. He worked for over 29 years at the National Geodetic Survey (NGS) of the National Oceanic and Atmospheric Administration, where he was promoted to the position of Chief Geodesist. In his federal career he developed accuracy standards, adjustment software, gravity and geoid models, GPS kinematic surveys, and vertical datum transformations. Dr. Milbert served on numerous federal technical and policy working groups and he was an alternate representative to the Senior Steering Group of the Interagency GPS Executive Board. He served for eight years on the joint Editorial Board for *Manuscripta Geodetica*/Bulletin Geodesique and the *Journal of Geodesy*. Dr. Milbert is a recipient of the Kaarina and Weikko A. Heiskanen Award, the NOAA Administrator's Award, the Department of Commerce Bronze Medal, and two Department of Commerce Silver Medals. He is a member of the American Geophysical Union, the International Association of Geodesy, and the Institute of Navigation. Dr. Milbert retired from government service in 2004 and pursues research in various geodesy topics.

APPENDIX

A.1. Datasheet for M 123

```

1      National Geodetic Survey,   Retrieval Date = NOVEMBER 25, 2011
TT2413 *****
TT2413 DESIGNATION - M 123
TT2413 PID - TT2413
TT2413 STATE/COUNTY- AK/DENALI BOROUGH
TT2413 USGS QUAD - HEALY C-4
TT2413
TT2413 *CURRENT SURVEY CONTROL
TT2413
TT2413* NAD 83(2007)- 63 43 21.43573(N) 148 57 43.76796(W) ADJUSTED
TT2413* NAVD 88 - 628.189 (meters) 2060.98 (feet) ADJUSTED
TT2413
TT2413 EPOCH DATE - 2007.00
TT2413 X - -2,426,212.599 (meters) COMP
TT2413 Y - -1,459,997.451 (meters) COMP
TT2413 Z - 5,696,668.363 (meters) COMP
TT2413 LAPLACE CORR- -8.38 (seconds) DEFLEC09
TT2413 ELLIP HEIGHT- 641.786 (meters) (07/17/09) ADJUSTED
TT2413 GEOID HEIGHT- 13.66 (meters) GEOID09
TT2413 DYNAMIC HT - 629.112 (meters) 2064.01 (feet) COMP
TT2413 MODELED GRAV- 982,033.7 (mgal) NAVD 88
TT2413
TT2413 HORZ ORDER - FIRST
TT2413 VERT ORDER - FIRST CLASS II
TT2413 ELLP ORDER - FOURTH CLASS I
TT2413
TT2413.The horizontal coordinates were established by GPS observations
TT2413.and adjusted by the National Geodetic Survey in July 2009.
TT2413
TT2413.The datum tag of NAD 83(2007) is equivalent to NAD 83(NSRS2007).
TT2413.The horizontal coordinates are valid at the epoch date displayed above.
TT2413.The epoch date for horizontal control is a decimal equivalence
TT2413.of Year/Month/Day.
TT2413
TT2413.The orthometric height was determined by differential leveling and
TT2413.adjusted in June 1991.
TT2413.WARNING-GPS observations at this control monument resulted in a GPS
TT2413.derived orthometric height which differed from the leveled height by
TT2413.more than one decimeter (0.1 meter).
TT2413
TT2413.The X, Y, and Z were computed from the position and the ellipsoidal ht.
TT2413
TT2413.The Laplace correction was computed from DEFLEC09 derived deflections.
TT2413
TT2413.The ellipsoidal height was determined by GPS observations
TT2413.and is referenced to NAD 83.
TT2413
TT2413.The geoid height was determined by GEOID09.
TT2413
TT2413.The dynamic height is computed by dividing the NAVD 88
TT2413.geopotential number by the normal gravity value computed on the
TT2413.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
TT2413.degrees latitude (g = 980.6199 gals.).
TT2413
TT2413.The modeled gravity was interpolated from observed gravity values.
TT2413
TT2413; North East Units Scale Factor Converg.
TT2413;SPC AK 4 - 1,083,325.355 551,279.408 MT 0.99993219 +0 55 50.2
TT2413;UTM 06 - 7,067,593.809 403,087.880 MT 0.99971501 -1 45 34.3
TT2413
TT2413! - Elev Factor x Scale Factor = Combined Factor
TT2413!SPC AK 4 - 0.99989959 x 0.99993219 = 0.99983179
TT2413!UTM 06 - 0.99989959 x 0.99971501 = 0.99961463
TT2413
TT2413 SUPERSEDED SURVEY CONTROL
TT2413
TT2413 NAD 83(2007)- 63 43 21.43619(N) 148 57 43.77233(W) AD(2007.00) 0
TT2413 ELLIP H (02/10/07) 486.989 (m) GP(2007.00)
TT2413 NAD 83(1992)- 63 43 21.43370(N) 148 57 43.78028(W) AD( ) 1

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TT2413 ELLIP H (06/20/05) 486.945 (m) GP() 4 1
 TT2413 NAVD 88 (07/17/09) 628.19 (m) 2061.0 (f) LEVELING 3
 TT2413 NAVD 88 (06/20/05) 474.1 (m) 1555. (f) GPS OBS
 TT2413 NGVD 29 (??/??/92) 626.505 (m) 2055.46 (f) ADJ UNCH 1 2
 TT2413
 TT2413.Superseded values are not recommended for survey control.
 TT2413.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
 TT2413
 TT2413_U.S. NATIONAL GRID SPATIAL ADDRESS: 6VVR0308767593(NAD 83)
 TT2413_MARKER: DD = SURVEY DISK
 TT2413_SETTING: 36 = SET IN A MASSIVE STRUCTURE
 TT2413_SP SET: SET IN BRIDGE
 TT2413_STAMPING: M 123
 TT2413_MAGNETIC: N = NO MAGNETIC MATERIAL
 TT2413_STABILITY: B = PROBABLY HOLD POSITION/ELEVATION WELL
 TT2413_SATELLITE: THE SITE LOCATION WAS REPORTED AS SUITABLE FOR
 TT2413+SATELLITE: SATELLITE OBSERVATIONS - May 31, 1989
 TT2413

| | | | | |
|--------|---------|------------|------------|-----------|
| TT2413 | HISTORY | - Date | Condition | Report By |
| TT2413 | HISTORY | - 1965 | MONUMENTED | CGS |
| TT2413 | HISTORY | - 19890531 | GOOD | JOA |

 TT2413
 TT2413
 TT2413 STATION DESCRIPTION
 TT2413
 TT2413'DESCRIBED BY COAST AND GEODETIC SURVEY 1965
 TT2413'1.8 MI W FROM MCKINLEY PARK.
 TT2413'1.8 MILES WEST ALONG THE MCKINLEY PARK HIGHWAY FROM THE RAILROAD
 TT2413'STATION AT MCKINLEY PARK, 0.2 MILE NORTHEAST OF THE ENTRANCE TO THE MT
 TT2413'MCKINLEY NATIONAL PARK HEADQUARTERS, 12 FEET SOUTHEAST OF THE CENTER
 TT2413'LINE OF THE HIGHWAY, SET IN THE TOP OF THE EAST CURB OF A CONCRETE
 TT2413'BRIDGE OVER ROCK CREEK, 15 FEET SOUTHWEST OF THE NORTHEAST END OF A
 TT2413'CONCRETE GUARDRAIL, AND ABOUT 0.6 FOOT HIGHER THAN THE ROAD.
 TT2413
 TT2413
 TT2413 STATION RECOVERY (1989)
 TT2413
 TT2413'RECOVERY NOTE BY JOHN OSWALD AND ASSOCIATES, LLC 1989
 TT2413'THE STATION IS LOCATED 0.2 KM (0.10 MI) NORTH EAST OF THE MCKINLEY
 TT2413'PARK HEADQUARTERS, 6.3 KM (3.90 MI) WEST OF THE PARKS HIGHWAY, IN A
 TT2413'BRIDGE OVER ROCK CREEK. OWNERSHIP, NATIONAL PARK SERVICE. TO REACH
 TT2413'THE STATION FROM THE INTERSECTION OF THE PARKS HIGHWAY AND THE
 TT2413'MCKINLEY PARK ROAD, DRIVE WEST ON THE ACCESS ROAD 6.3 KM (3.90 MI) TO
 TT2413'THE ROCK CREEK BRIDGE. STATION IS ON THE LEFT. THE STATION IS A 7.6
 TT2413'CM BRASS DISK SET IN THE SOUTH EAST BRIDGE ABUTMENT OF THE ROCK CREEK
 TT2413'BRIDGE, LOCATED 3.6 M (11.8 FT) SOUTHEAST OF THE CENTERLINE OF THE
 TT2413'ROAD, 4.5 M (14.8 FT) SOUTHWEST OF THE END OF THE ABUTMENT, 0.3 M (1.0
 TT2413'FT) NORTHEAST OF A WITNESS DECAL ON THE RAIL OF THE BRIDGE. NOTE, THE
 TT2413'MAGNETIC PROPERTIES OF THE STATION ARE UNKNOWN.

A.2. Output for the GPS2828 Case Study

```
C>harnstat
  program harnstat -- 2012jan18
input  80/86 file  (input) : 8086-harn.txt
input  bbook file  (input) : bbookharn.txt
      45 points loaded
1001    0.6    1.5    0.0
1002    0.0   -3.7    1.0
1003    1.2   -4.6   -2.0
1004    0.3   -0.6    0.0
1005    0.9    0.6    1.0
1006    0.3   -3.7   -1.0
0001    0.0   -0.6   -2.0
1007   -0.9   -4.6    0.0
1008    0.0   -0.3    0.0
1009   -0.9   -3.4    4.0
1010    0.3    0.9    0.0
0002   -7.7   17.9    7.0
1011    0.0    2.8    1.0
1012    0.0    1.2    0.0
1013    0.3    0.0    0.0
1014    0.0   -2.8    3.0
1015    0.6   -0.6    1.0
1016   -2.8   -5.6    2.0
1017   -2.2   -5.3    6.0
1018    0.3    0.0    1.0
1019   -0.9   -4.6    3.0
1020    0.0    2.2    1.0
1021   -2.2   -4.9    4.0
1022    0.0   -1.2    2.0
1023    0.0    0.0    0.0
1024   -1.2   -3.7    5.0
1025   -0.6    3.7    1.0
1026   -1.2   -4.0    7.0
1027    0.0   -0.6    0.0
1028   -0.9    7.1    2.0
1030    0.9   -4.3   -3.0
1029   -0.3   -2.2    4.0
1031    0.3    1.5    0.0
1032    1.2   -0.3   -1.0
1033   -0.3    4.9    2.0
1034    0.3    0.9    1.0
1035   -2.2   -4.9    3.0
1036    0.0   -4.0   -1.0
1037    2.5   -6.5   -5.0
1038    0.0   -0.3    0.0
1039    0.6    0.0    0.0
1040    0.0    0.0   -1.0
1041    0.3   -0.6    1.0
1042    0.0    1.5    2.0
1043   -0.3    3.1    1.0
      45 points processed
ave:    -0.3   -0.6    1.1
rms0    1.5    4.2    2.6
rms:    1.5    4.1    2.4
```